

## TO UNDERSTANDING SOME INTRIGUING SEGREGATION PHENOMENA IN RAPID GRAVITY FLOWS OF PARTICULATE SOLIDS ON A VIBRATED ROUGH CHUTE

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**Abstract:** The research results on segregation in vibrated granular media obtained and published so far are discussed in the paper. The most intriguing segregation phenomena are revealed in the course of discussion. The analysis of the segregation phenomena without taking into account any structural characteristics of vibrated granular media is defined as the main reason for their controversial explanations. A method of determination of granular temperature dependence on the depth coordinate of a vibrated bed of particles is proposed to forecast structural characteristics on the basis of a granular medium state equation.

### Symbols

|   |   |
|---|---|
| $b = (p/(6(1-e)))^{0.33}$ – geometrical parameter;  | $\varepsilon$ – fraction of void volume, $\text{m}^3 \cdot \text{m}^{-3}$ ;                                     |
| $d$ – diameter of particles, m;   | $\varepsilon_0$ – fraction of the void volume of dense packed particles, $\text{m}^3 \cdot \text{m}^{-3}$ ;     |
| $F$ – collision frequency of particles, $\text{s}^{-1}$ ;   | $\lambda$ – experimental coefficient of the tangent velocity loss at particle collision;                        |
| $p(y)$ – analogue of hydrostatic pressure, $\text{N} \cdot \text{m}^{-2}$ ;   | $\mu$ – friction coefficient;   |
| $k$ – restitution coefficient at collisions;  | $v_g$ – granular temperature generated by the gravity force, $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ ; |
| $Q$ – quasithermal flux;  | $v_v$ – granular temperature generated by vibration, $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ ;         |
| $s$ – mean distance between particles, m;   | $\chi$ – coefficient of the granular medium state equation;   |
| $V'$ – fluctuation component of the particle velocity, $\text{m} \cdot \text{s}^{-1}$ ;                                     | $\omega$ – vibration frequency, $\text{s}^{-1}$ .   |
| $y$ – Cartesian coordinate along the bed depth, m;  |   |
| $\bar{\varepsilon} = (\varepsilon - \varepsilon_0)/(1 - \varepsilon)$ – flow dilatation, $\text{m}^3 \cdot \text{m}^{-3}$ ; |   |

Segregation in particulate solids on a vibrated bottom reveals itself as one of the most intriguing phenomena both in particle technology science and industry [1, 2].

A close scrutiny of the literature on segregation during vibration of granular materials shows that many theoretical and experimental investigations have been carried out to study the features of particle behavior in horizontal vibrated beds [1 – 11]. It takes place in spite of high practical interest to vibration in shear flow conditions, when the vibrorheological and shear effects are imposed at each other.

However, even in this ordinary case of granular matter vibration there are many intriguing segregation phenomena, which so far remain as objects of discussion in terms of their physical mechanisms.

Brown [3] suggested that the mechanism of segregation under vibration occurs because of locally nonuniform packing conditions in the neighborhood of large and small particles.

Williams [4] considered inter particle percolation to be the prime factor in segregation. This author investigated size segregation by varying the frequency of vibration at constant amplitude and showed that the larger particles can always be made to rise. He suggested that if both frequency and amplitude could be varied, the behavior of the system might be determined by the value of vibration acceleration.

Fattuhi showed that segregation increased with increase in amplitude, however, at higher frequencies segregation was reduced for various shapes and sizes of particles. In order to study the phenomenon of segregation in granular systems due to vibration, Ahmad and Smalley [5] carried out some experiments on migration of a large particle through a dry granular bed under different vibration conditions. They found out that at a constant frequency of vibration, segregation increased with increase in acceleration; however, with increasing frequencies segregation was correspondingly reduced.

At present there are many new special terms designating the physical segregation mechanisms in vibrated granular materials, such as: convection, condensation, collective reorganization, air – driven and void-filling segregation and others. However, in the course of studying the segregation phenomenon during granular matter vibration a lot new mysterious effects have been found which need to be understood now. The most intriguing between them are the “Brazil Nut Problem” – BNP and related with it Reverse Brazil Nut problem – RBNP. These problems consist in respectively the following questions: why the Brazil nuts are on top; why all the Brazil nuts are not on top [1, 6]. So far these questions are the discussion matter in order to reveal the underlying physical mechanisms of the BNP and RBNP.

Knight et al. [7] investigated the segregation of granular materials while they are being shaken. They believed that the particle segregation in this case takes place due to the global convection currents in the bed.

Barker and Mehta [8] state that when considering segregation in accordance with particle size in vibrated granular materials it is necessary to take into account two mechanisms: convection and diffusion. Thus, the main mechanism for size segregation is the competition between independent-particle and collective reorganization. The competing domains of amplitude and frequency need to be investigated experimentally in order to find better control parameters for vibrating beds.

Other researches [9] consider the arching as the main mechanism causing the BNP. Rosato et al [10] believe that the main mechanism promoting the large particle ascension in a shaken bed of granular matter is the void-filled volume beneath large ascending particles.

Liffman and co-authors [11] observed inertia as the underlying mechanism explaining the BNP.

It seems to be that a considerable contribution to the final understanding the physical mechanisms of this elusive granular matter problem (BNP, RBNP) was made with experimental research carried out by Huerta and Ruiz-Suarez [2]. They studied the segregation of single large spheres in a granular bed of small ones under vertical vibrations. In their experiments the rise and sink times of large spheres were measured as functions of relative values of its density, diameter and sinkage depth in the bed for two different conditions of sinusoidal vibrations. These conditions were provided at the same vibration acceleration  $a_v = 3g$  and differ from each other by vibration frequency 5 and 50 Hz.

Basing on the experimental results they have concluded that there are only three physical segregation mechanisms of large or small, heavy or light particle in a vibrated bed of granular materials: inertia, convection and buoyancy (sinkage), which exist at vibration accelerations more than 1g. Thereat, the first two mechanisms always present at low frequencies and inertia dominates when the relative particle density is greater than one while convection prevails when the relative density is less than one.

The buoyancy and sinkage take place at high frequencies when convection is suppressed and inertia is negligible. Thereat, the light particle having the relative density less than one ascends and heavy particle its relative density greater than one sinks in a vibrated bed of granular material.

Thus, the authors [2] explain the BNP as a result of action of the inertia and convection mechanisms at low vibration frequencies. In their opinion more dense large particles ascend in a vibrated bed under inertia forth action, while the light particle ascension is explained by global convection when there is ascending flow the central part of a vibrated container and falling flow – in its peripheral parts.

However, these results can not explain a lot of intriguing phenomena some of them were observed by the authors [1, 2]:

- large light particle having the relative density less than one can not ascend in the horizontal vibrated bed of small granulate this particle is located at the bed bottom;
- all the Brazil nut (heavy large particles) are not on top of vibrated granular bed of nut mixtures (Reverse Brazil Nut Problem, RBNP): heavy large particles are concentrated on top and beneath of vibrated bed of nonuniform particulate solids;
- the more the large particle density differs from the particle density of a vibrated bed the faster this particle ascends towards free bed surface. The ascension velocity of large particle at these conditions is minimum, when its density is equal to one.

It seems to be that the reason of this situation is analyzing the segregation phenomena without any taking into account structural characteristics of a vibrated bed. Basing on the results of experimental and analytical research [12, 13] we can assume that spatial structural characteristics of granular media are very important for segregation dynamics.

Forecasting structural and kinematic characteristics in vibrated beds of granular media is a rather difficult problem because of their changeable and elusive character. Experimentally this problem is solved by the methods using several penetrating radiations: magnetic resonance imaging – MRI; electric – impedance tomography – ERT; X – ray tomography and others. However these methods are very problematic when the local characteristics must be determined.

Recently, we suggested [13] a method allowing determination of structural and kinematic characteristics in rapid gravity flows of granular materials on a vibrated rough chute. The method assumes an analogy between particulate medium under conditions of relatively large component of fluctuation velocity of particles and dense gas.

This analogy is established as the interrelationship between dilatation  $\bar{\epsilon}$ , normal pressure  $p$  and granular medium temperature  $v$  during two dimensional vibrated shear flow of nonelastic cohesionless particles as follows [13]

$$p(y)\bar{\epsilon}(y) = \chi v(y) = \chi(v_g(y) + v_v(y)), \quad (1)$$

where:  $v_g(y)$  and  $v_v(y)$  are the granular temperatures generated by the gravity forces and vibration respectively;  $p(y)$  is the hydrostatic pressure analogue;  $\bar{\epsilon}(y)$  is the flow dilatation;  $\chi$  is the coefficient of the particulate medium state equation.

In order to determine the structure characteristics in a steady state vibrated flow with due regard to eq. (1), it is necessary to establish the granular medium temperature as a function of the  $y$  – coordinate perpendicular to the shear flow direction and vibrating bottom.

The granular medium temperature is defined as an analogue of the kinetic energy of particles caused by their mutual displacement. The granular medium temperature  $v_g(y)$  generated by the gravity forth was defined as the following elementary types of mutual displacements of particles:

- the energy caused by the relative velocity of particles to shear direction;
- the energy depending on the chaotic fluctuation of particles;
- the energy caused by transversal mass transfer of particles.

In a steady state the quasithermal flux generated by vibrating bottom towards particle bed is proportional to the bed pressure on the bottom.

The flux value is calculated with taking into account physical and mechanical properties of the bottom surface and particles, vibration parameters also structural and kinematic flow characteristics [13].

The quasithermal flux was determined as the sum of normal  $Q_n$  and tangent  $Q_t$  components of power action and energy dissipation  $Q_d$  flux

$$Q(h) = Q_n(h) + Q_t(h) - Q_d(h). \quad (2)$$

Earlier, supposing the quasithermal flux is proportional to the concentration of particles and their collision frequency with the bottom, we had determined [13] the interrelationship between the thermal flux and granular temperature in the following way

$$Q(y) = b^{-2}(y)d^{-2}(y)v_v(y)F(y) = b^{-2}(y)\frac{d^{-2}(y)m(V'(y))^2}{2}\frac{V'(y)}{s(y)}, \quad (3)$$

where  $F(y) = V'(y)/s(y)$  is the collision frequency of particles.

At this approach it is principally important to define correctly the fluctuation velocity  $V'$ , generated by the bottom vibration, as a function of a flow depth perpendicular to the bed bottom.

In order to find the  $V'(y)$  dependence we suppose that the rate of fluctuation velocity reduction from bottom to free surface is proportional to the energy fraction dissipated at particle collisions and their frequency. Obviously, the collision frequency may be determined by means of multiplication of linear concentration of particles  $(bd)^{-1}$  and their fluctuation velocity. Then, we can express the rate of fluctuation velocity reduction in the following way

$$\frac{dV'(y)}{dy} = -e_c^{0.5}(b(y)d(y))^{-1}V', \quad (4)$$

where  $e_c$  is the kinetic energy part dissipated at particle collision which can be calculated as follows [14]

$$e_c = \frac{1}{2}(1-k^2) + \frac{\pi}{32}\mu^2(1+k)^2\frac{\pi\lambda^2}{8} - \mu(1+k) - \frac{\pi}{8}\mu\lambda(1+k).$$

The extent 0.5 of  $e_c$  is explained by parabolic dependence between kinetic energy and fluctuation velocity.

After division of variables in (4) and integration ranging from  $V'(h)$  to  $V'(y)$  and from  $h$  to  $y$  we can obtain the following dependence:

$$\int_{V'(h)}^{V'(y)} \frac{dV'(y)}{V'(y)} = \int_h^y -e_c^{0.5}b(y)^{-1}d(y)^{-1}dy; \quad (5)$$

$$V'(y) = V'(h) \exp\left(-\int_h^y e_c^{0.5}b(y)^{-1}d(y)^{-1}dy\right). \quad (6)$$

In this equation  $V'(h)$  is the fluctuation velocity of particles contacting immediately with a vibrated bottom, which can be calculated in compliance with (3) as follows

$$V'(h) = b(h)d(h)\left(\frac{2Q(h)}{m\omega}\right)^{0,5}.$$

Because of structural nonuniformity of vibrated granular flows eq. (6) can be integrated only numerically.

The structural characteristics in a vibrated granular flow on a rough chute are determined on the basis of eq. (1) with the iteration procedure described in [15].

The dependence obtained (6) allows to make qualitative supposition that the granular temperature in a vibrated bed of particles on a rough chute will increase exponentially towards the bed bottom. Because of that the solid phase concentration near the bottom will decrease the elevation of the concentration gradient which becomes the reason for migration effect [16]. The migration effects may be used for understanding some of the intriguing segregation phenomena in vibrated granular media. Obviously, it is necessary to check the obtained mathematical description adequacy experimentally.

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## **К пониманию некоторых загадочных явлений сегрегации в быстрых сдвиговых потоках твердых частиц на шероховатом скате**

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**Ключевые слова и фразы:** вибрация; гравитационный поток; миграция; сегрегация; сдвиговые потоки.

**Аннотация:** Рассмотрены опубликованные к настоящему времени результаты исследований сегрегации в виброожиженных зернистых средах. В процессе обсуждения выявлены малоизученные эффекты сегрегации. Основная причина спорных объяснений эффектов сегрегации при их анализе – отсутствие информации о структурных характеристиках виброожиженных зернистых сред. Предложен метод определения температуры зернистой среды как функции координаты толщины вибросжиженного слоя, необходимый для прогнозирования структурных характеристик с использованием уравнения состояния зернистого материала.

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## **Zum Verständnis einiger geheimnisvoller Erscheinungen der Segregation in den schnellen Schiebeströmen der festen Teilchen auf dem unebenen Abhang**

**Zusammenfassung:** Im Artikel werden die zur Gegenwart veröffentlichten Ergebnisse der Forschungen der Segregation in den vibroverflüssigten körnigen Umgebungen besprochen. Im Laufe der Erörterung werden die für diesen Fall geheimnisvollsten Effekte der Segregation gezeigt. Als Hauptgrund der strittigen Erklärungen der Effekte der Segregation bei ihrer Analyse wird die Abwesenheit der Informationen über die strukturellen Charakteristiken der vibroverflüssigten körnigen Umgebungen genannt. Es wird die Methode der Bestimmung „der Temperatur der körnigen Umgebung“ wie die Funktion der Koordinate der Dicke der vibroverflüssigten Schicht vorgeschlagen. Diese Methode ist für die Prognostizierung der strukturellen Charakteristiken unter Ausnutzung der Angleichung des Zustandes des körnigen Materials notwendig.

## **Vers une compréhension de certains phénomènes énigmatiques de la ségrégation dans les courants des particules solides sur une pente rude**

**Résumé:** Dans l'article sont discutés les résultats publiés pour le moment actuel des études de la ségrégation dans les milieux vibroliquéfiés. Lors de la discussion sont révélés les cas les plus énigmatiques de l'effet de la ségrégation. La raison la plus discutable de l'explication de ces effets est l'absence de l'information sur les caractéristiques structurelles des milieux vibroliquéfiés. Est proposée la méthode de la définition de la «température du milieu vibroliquéfié» comme fonction de la coordonnée de l'épaisseur de la couche vibroliquéfiée nécessaire pour la prévision des caractéristiques structurelles avec l'emploi de l'équation de l'état du matériau granuleux.

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