## MODELING OF INDUCTION HEATING DEVICES IN EXAMPLE OF INDUCTION HEATING PLATENS OF VULCANIZATION PRESSES

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**Key words and phrases**: finite integral transformations method; heating platen; induction heating; method of thermal calculation; vulcanization press.

**Abstract:** This paper is focused on the modeling of induction heating platens of vulcanization presses. Main statements of platen's design procedure are depicted, which is based on solving three-dimensional non-stationary heat conduction equation by using finite integral transformations method. General questions of platen's design are analyzed.

Heating of technological equipment, heat treatment, welding, brazing, soldering and melting are applications of induction heating in industry. It is a process of heating of current-conducting materials (metals usually) due to electromagnetic induction, which produces eddy currents (Foucault currents) leading to heating of materials. High speed of heating, high power densities, small heating time, easy automatization and control, clean and safe operating conditions are the main benefits of this type of heating. In general, induction heating device consists of three main parts: AC power supply, inductor and heating object called loading.

We should emphasize the complexity of induction heating. For its precise description we should take into account the sequence of conjugate problems: electromagnetic, thermal, hydrodynamic, mechanic and metallurgic [1]. The first three of them are using in engineering calculations. The main problem is electromagnetic, because it's solving is taken as a volume initial condition for thermal analysis.

We discuss special features of mathematical modeling of induction heating devices in example of heating platens of vulcanization presses for producing general mechanical rubber goods. Thermal problem is the most important, because the quality of production is directly depends on temperature field. That's why in this paper we take into account only thermal problem.

At least two factors are complicating designing of platens. Firstly, physicalmechanical, thermal and technological properties of platen's materials are constrain the obtaining of needed temperature distribution on the working surface of a platen. Secondly, the accuracy of existent methods of calculation is unsatisfactory and long time is needed for processing the calculations.

Mentioned factors considerably complicate the optimization of structural and technological characteristics of a platen. However, we should attend to quality of platens because vulcanization is the ending process in rubber industry manufacturing.

Currently we use a method of mathematical modeling and calculation of platens which is based on the approximate solving of analytical conductive equation. The main statements of this approach are described in [2].

In this method the differential conduction equation is used as a main functional dependence. Heating of induction platen is described by three-dimensional non-stationary differential conduction equation with partial derivatives.

$$\frac{\partial T}{\partial \tau} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{q(x, y, z, T_{av})}{c\rho},$$
(1)

where  $T = T(x, y, z, \tau) - a$  platen temperature in a point of its volume with coordinates (x, y, z) at time  $\tau$ ; a - a platen material coefficient of thermal diffusivity,  $a = \lambda/(c \cdot \rho)$ ;  $\lambda - a$  platen material coefficient of thermal conductivity, Wt/(m·K); c - a platen material thermal conductivity, J/(kg·K);  $\rho - a$  platen material density, kg/m<sup>3</sup>;  $T_{av} - a$  platen average temperature in the moment of time  $\tau$ , °C,

$$T_{\rm av} = T_{\rm av}(\tau) = \frac{1}{hsl} \int_{0}^{h} \int_{0}^{s} \int_{0}^{l} T(x, y, z, \tau) \, dx \, dy \, dz;$$

 $q(x, y, z, T_{av})$  – internal heat generation intensity dependence of coordinates and average platen temperature, Wt/m<sup>3</sup>,

$$q(x, y, z, T_{av}) = \begin{cases} Q_{ij}(T_{av})/v_{pj}, \text{ if } (x, y, z) \in v_{pj}, j = 1, ..., n_i; \\ 0, \text{ else}; \end{cases}$$

 $Q_{ij}(T_{av})$  – power of inductor with *j*-index at average platen temperature, Wt;  $v_{pj}$  – volume of internal heat generation by inductor with *j*-index, m<sup>3</sup>.

Initial condition for equation (1) is

$$T(x, y, z, 0) = T_0,$$
 (2)

where  $T_0$  – ambient air temperature, °C.

Boundary conditions:

$$\lambda \frac{\partial T(0, y, z, \tau)}{\partial x} - \alpha_1 (T(0, y, z, \tau) - T_0) = 0;$$
  

$$\lambda \frac{\partial T(l, y, z, \tau)}{\partial x} + \alpha_1 (T(l, y, z, \tau) - T_0) = 0;$$
  

$$\lambda \frac{\partial T(x, 0, z, \tau)}{\partial y} - \alpha_2 (T(x, 0, z, \tau) - T_0) = 0;$$
  

$$\lambda \frac{\partial T(x, s, z, \tau)}{\partial y} + \alpha_2 (T(x, s, z, \tau) - T_0) = 0;$$
  

$$\lambda \frac{\partial T(x, y, 0, \tau)}{\partial z} - \alpha_3 (T(x, y, 0, \tau) - T_0) = 0;$$
  

$$\lambda \frac{\partial T(x, y, h, \tau)}{\partial z} + \alpha_4 (T(x, y, h, \tau) - T_0) = 0,$$
  
(3)

where  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  – complex heat-transfer coefficients of a platen's edges with fastening plates, the butts without fastening plates, a platen working surface and the cover surface respectively, Wt/(m<sup>2</sup>·K),

$$\alpha_1 = \alpha(t_{\rm ed}) + \frac{\lambda_{\rm p}}{l_{\rm p}} \frac{n_{\rm p} h_{\rm p} s_{\rm p}}{hs}, \quad \alpha_2 = \alpha(t_{\rm ed}), \quad \alpha_3 = B_{\rm b} \alpha(t_{\rm ed}), \quad \alpha_4 = B_{\rm u} \alpha(t_{\rm ed}),$$

where  $\alpha(t_{ed})$  – heat-transfer coefficient of a platen's surfaces to ambient air by convection and radiance, which can be defined by recommendations in [3],  $\alpha(t_{ed}) = \alpha_{cond}(t_{ed}) + \alpha_{rad}(t_{ed})$ ,  $t_{ed}$  – a platen side temperature, °C;  $n_p$  – number of fastening plates in a platen butt;  $\lambda_p$  – a platen fastening plates coefficient of thermal conductivity, Wt/(m·K);  $s_p$ ,  $h_p$ ,  $l_p$  – width, height of plates section and their out-of-platen length part ширина, m;  $B_b$ ,  $B_u$  – coefficients obtaining different conditions of heat-transfer of bottom and upper platen surfaces.

For solving (1) - (3) problem we made three assumptions.

1) Heat transfer coefficients  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  of all surfaces of platen and inductors power  $Q_{ij}$ ,  $j = 1, ..., n_i$ , where  $n_i$  – quantity of inductors, are not depend on temperature during rated periods of time.

2) As shown in [4], if required heating temperature is not exceeding Curie temperature (750 °C for steel), then it is possible to solve the problem of determination of evolved in inductors' slots power independently from the problem of heat-transferring in the volume of a platen.

For this reason we made an assumption that heat-flux from each inductor is homogenous in a volume of slot. Also we use empirical method for determining the  $Q_{ij}$  values of fixed load temperature, which is based on experimental researches of induction heating of ferromagnetic steel [4]. Characteristics of inductors and organic-silicate compound are corresponding to characteristics of a platen material.

The process of heat transfer in a platen under the fixed values of  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  and  $Q_{ij}$  is iterative. Therefore we calculate these parameters in rated periods of time in which they are constant. Initial value for all solutions except first iteration is temperature distribution in a platen appropriated to the moment of finishing of previous time period. Justified calculating of rated periods of time is a main problem during the realization of this approach.

Experimental and computational researches determine that during the initial period of a platen heating the change of speed of heat-transfer coefficients is higher than the change of speed of inductors' power changing. In final period it is vice versa. Therefore it needs to solve the problem of choosing time-step in combined evaluation of heattransfer coefficients and inductors' power.

The following method is suggested for solving this problem. For initial moment of time evaluation of average temperatures of all surfaces is produced. Respectively to these values the meaning of heat-transfer coefficients and inductors' power is evaluated. Then the time of heat is increased by defined value of time-step and recalculation of heat-transfer coefficients and inductors' power is take place. Time of heat is increasing until the difference between initial and calculated values of mentioned beyond parameters exceeds the predetermined accuracy. Evaluated by that method value of heating time will become initial time for the next iteration.

The finite integral transformations method is used as method of analysis of (1) - (3) problem [5]. The choice of this method is caused by possibility of obtaining analytical solving in case of nonuniform boundary conditions with application of unified methods of coordinates elimination. The use of numerical methods for computing assigned task will be analyzed in the future researches.

According to the described method on example of induction heating platen  $500 \times 410$  mm with four rectangular inductors by heating time 32.8 min (1968 s) under the condition of 10 % value of accuracy were taken results, represented in Fig. 1.

As shown in the Fig. 1, 14 time-steps were obtained for chosen calculation accuracy. Similarly for 15 % accuracy were obtained 10 time-steps, 20 % - 8 time-steps, 25 % - 6 time-steps.



of a platen heating time  $\tau$ , accuracy 10 %

In our opinion the difference equal 20 % between previous and next values of heat-transfer coefficients and inductors' power according to the finite integral transformations method is optimal for realization of thermal calculations of platens. This accuracy is comparable to accuracy of heat-transfer coefficients determining according to conduction criterion equations [3] and comparable to accuracy of determining inductors' power method developed by professor A.B. Kuvaldin [4]. As a result, acceptable computation time of about 20 min is achieved with satisfactory accuracy of thermal calculations.

We would like to mention that this method does not take into account the electromagnetic side of induction heating, because evaluation of inductor power is maintained by empirical engineering method, i.e. without solving Maxwell equations and determining of magnetic induction distribution inside slots.

Usually the quality of existent heating platens estimated by the degree of homogeneity of a temperature field on its working surface. It is considered that for modern platens the difference of temperature on working surface should be  $\pm 1...2$  °C during the process of vulcanization. However, it is necessary to take into account special requirements to the formed temperature field for all specific cases of induction heating applications and to analyze appropriateness of using existing technologies usage [6].

It is possible to draw a conclusion that tendency to design of heating platens with only homogenous temperature field on the working surface is incorrect. That approach includes the following methodological errors.

Firstly, it is not considered that assortment of produced mechanical rubber goods is wide both in the type sizes, and processed rubbers as well as using technologies. From the point of view of rubber production engineering it is necessary to obtain homogenous temperature field on internal surface of mold not on the working surface of a platen.

Secondly, according to the numerical calculations of different induction heating devices [6–8], it is impossible to obtain a homogenous magnetic field inside the platen volume. Therefore from the viewpoint of induction heating physics it is inaccessible to form homogenous temperature field of the platen surface.

Considering the aforesaid, we can draw a conclusion that the problem of obtaining the temperature field which will be well-corresponded to the manufacturing production is actual. The problem of acquisition of that kind of field is formulated as follows.

For induction heating platen under the set sizes (length l, width s, height h) of the platen; materials of platen, cover and inductors; parameters of electric network (voltage U, frequency f), diameters of inductors wires it is necessary to find such number of inductors  $n_i$ , length  $l_j$  and width  $s_j$  of each inductor, center of each inductor coordinates [ $x_{cj}$ ;  $y_{cj}$ ], number of coils of each inductor  $n_j$ , width  $b_j$  and depth  $z_j$  of all slots, that by control thermocouple temperature achievement of  $t_c = t^* \pm \varepsilon$  upon the ending of set time the calculated and set temperature profiles will have minimal differences

$$\sqrt{\frac{1}{k+m} \sum_{q=1}^{k} \sum_{p=1}^{m} \left| T_{qp} - T_{qp}^{*} \right|} \to \min, \qquad (4)$$

where  $T_{qp}$  – evaluated temperature in the working surface point with coordinates  $(qh_l; ph_s)$ , °C;  $h_l$ ,  $h_s$  – discrecity of set temperature profile by length and width of a platen;  $T_{qp}^*$  – set heating temperature of a platen in the point of working surface, °C;

The search of a minimum of function (4) is realized by the following limitations:

1) evaluated temperature profile in all points of the working surface should have difference from the set profile within allowed error

$$\left|T_{qp} - T_{qp}^{*}\right| \le \Delta T^{*}, \ q = 1,...,k, \ p = 1,...,m,$$
(5)

where  $\Delta T^*$  – maximum error on a working surface, °C;

2) difference between set ending heating temperature  $t^*$  and evaluated temperature in the place of control thermocouple  $t_c$  should not exceed demanded accuracy  $\varepsilon$ 

$$\left|t_{\rm c}-t^*\right| \le \varepsilon; \tag{6}$$

3) limitation on a total average platen power  $Q_p$ :

$$\sum_{j=1}^{n_{i}} \mathcal{Q}_{ij} \le \mathcal{Q}_{p} ; \tag{7}$$

4) limitations on inductors' sizes:

$$l_{ij} \in [l_{i*}; l_i^*], \ s_{ij} \in [s_{i*}; s_i^*],$$
(8)

where  $l_{i*}$ ,  $s_{i*}$  – minimum length and width of inductors respectively;  $l_i^*$ ,  $s_i^*$  – maximum length and width of inductors respectively;

5) limitation on power factor and performance of induction heating:

$$\cos \varphi \ge \cos \varphi_{\min}; \tag{9}$$

$$\eta \ge \eta_{\min},\tag{10}$$

where  $\cos \varphi_{min}$ ,  $\eta_{min}$  – minimum power factor and performance of induction heating respectively.

That is task decomposed on the number of conjugate tasks which are represented on Fig. 2.



The number of inductors, their relative positioning and inclusion scheme are set on the first level. Evaluation of inductors' power and number of coils is implemented on the second level. Inductors' length, width and their coordinates of centers are calculated on the third level. That division of variables on levels is caused by different influence of these variables on obtained temperature field.

Due to that we would like to bring attention to existent conditions in the sphere of modeling induction heating devices. Analysis of publications [7, 9, 10] allows us to draw a conclusion about lack of attention to the problem of optimization of induction heating devices. On one side, authors of mentioned papers try to build precise mathematical model of processes taken place during the heating, take into account non-linear dependency of thermal-physics and electromagnetic properties, use modern computer technologies for carrying out calculations. But their work is limited by only obtaining temperature, magnetic induction and current density distribution and by discussion of the results. As usual that discussion is just description of graphs and repeating well-known facts about advantages of mathematical modeling above physical modeling and optimization prospects. We don't know publications where authors represent problem definition in formalized a type.

Experiments of efficiency comparison of platens' with induction and resistive (tubular electric heating elements) heaters in "ARTI-Zavod" plant have been conducted for verification of created mathematical model.

The experiment was done at air temperature  $T_0 = 12$  °C on the specially made table in the plant's department of energy. Heating platen with sizes l = 500 mm, s = 410 mm, h = 70 mm with 4 rectangular inductors  $172 \times 127$  mm in slots with  $25 \times 25$  mm crosssection was established by cover downwards leaning on three supporting screws with spherical head. Screws were arranged by edges of triangular with base 300 mm and height 260 mm. Horizontal position of platen was obtained by these screws. The distance between table and platen surface was 150 mm. Inductors were connected serially. Copper wire in diameter of 1,8 mm was used for inductors. Number of coils was 60 in each inductor, their total power by  $T_0$  temperature was 5,35 kWt.

Chromel-copel thermocouples in thermoelectrode diameter of 0,5 mm which were made and calibrated by control equipment service were used for thermal measuring. Four working thermocouples were located at the platen's corners from 50 mm of edges, fifth thermocouple was in the center. Blind holes by diameter of 5,5 mm with depth 5 mm were drilled in platen at the locations of thermocouples from the working surface. Aluminum plugs made of wire by diameter of 5 mm were inserted and clenched inside these holes. Thermojunction by diameter of 1,5 mm was calked in plugs by 2 mm depth. Control thermocouple was placed in hole on the short platen end and located in 16 mm depth from the working surface and by 90 mm from short end and by 130 mm from long end. Scheme of thermocouples location during the experiment is shown on the Fig. 3.

Thermocouples were connected to the A-565-003 device, temperature on the working surface measuring was provided by digital contact thermometer TK-5.03, electric parameters of a platen were controlled by measuring complex K505 1621-75. Time of heating was controlled by stopwatch, end heating temperature was 170 °C.

Heating time to end temperature in the experiment was 32,8 min (1968s). Results of consistent evaluating of the (1) – (3) task for rated periods of time in which changing of heat-transfer coefficients and inductors' power were not exceeding 20 % and their comparison to experimental data are represented on Fig. 4.

As we can see from the Fig. 4, results of developed method applied for solving task (1) - (3) are well-matched with experimental data received on the real manufacture.



Fig. 3. Scheme of thermocouples location in the experiment



for control thermocouple temperature

Experimental data also let to evaluate the accuracy of calculating of inductor's power by set platen temperature. That method was proposed in [4]. The results of a platen power measurements during experiment and their comparison with calculated data are shown on Fig. 5. Computing error in compare of experimental data did not exceed 3 %.



We would like to mention the main problems during task (1) - (9) solving. Firstly, there is a necessity for taking into account of non-linear changing of electromagnetic and thermo physics properties of a platen and inductors materials. The least studied characteristics which influence on the whole calculating process is magnetic inductivity. It depends essentially on electromagnetic field density. Existent data from [4] has empirical character and not suitable for engineering calculations of heating platens.

Secondly, it is needed to create reliable optimization algorithm which will

provide calculations automatically. Authors of paper [11] fairly mentioned that for any induction heating device calculation task is typical to find compromise between evaluation accuracy and time finding. That definition is actual to all methods of calculating.

The algorithm of solving task (1) - (3) is realized in Mathcad system on PC with dual-core CPU with frequency of single core 2,7 MHz and 2 Gb RAM. Approximately 25 min were needed for computing temperature field of 2091 points of the platen which construction was described in [2]. For providing optimization of calculations this time will increase in proportion to total quantity of variables. Therefore preliminary finding of these variables definitional interval for minimization computational time and achievement of set accuracy is an actual problem.

Solving of these problems is the future work for modeling and calculation on induction heating platens of vulcanization presses for producing general mechanical rubber goods using finite integral transformations methods.

The present research is executed in the frame of state contract  $N_{0.02}$  02.740.11.0624 of the Federal program "Scientific and scientific-pedagogical staff of the innovative Russia for the years 2009–2012".

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## Моделирование устройств индукционного нагрева на примере индукционных нагревательных плит вулканизационных прессов

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Ключевые слова и фразы: вулканизационный пресс; индукционный нагрев; метод конечных интегральных преобразований; методика теплового расчета; нагревательная плита.

Аннотация: Рассмотрены вопросы моделирования индукционных нагревательных плит вулканизационных прессов. Приведены основные положения методики расчета плит, основанной на решении трехмерного нестационарного уравнения теплопроводности методом конечных интегральных преобразований. Проанализированы вопросы проектирования плит.

## Modellierung der Anlagen der Induktionserwärmung am Beispiel der Induktionsheizherde der Heizpressen

**Zusammenfassung:** Es sind die Fragen der Modellierung der Induktionsheizherde der Heizpressen betrachtet. Es sind die Grundlagen der auf der Lösung der dreidimensionalen nichtstationären Gleichung der Wärmeleitung durch die Methode der Endintegralwandlungen gegründeten Methodik der Berechnung der Herde angeführt. Es sind die Fragen der Herdeprojektierung analysiert.

# Modélage des dispositifs du chauffage inductif à l'exemple des plaques de chauffage inductif des presses de vulcanisation

**Résumé:** Sont examinés les problèmes du modélage des plaques du chauffage inductif des processus de vulcanisation. Sont citées les essentielles formules de la méthode du calcul des plaques fondées sur la solution de l'équation non stationnaire à trois dimensions du transfert de la chaleur par la méthode des transformations finales intégrales. Sont analysées les questions de la conception des plaques.

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